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ABSTRACT

This chapter describes a research design model that utilizes both theory and common, core classroom conditions in order to create and investigate new instructional strategies. Transformative and conjecture-driven teaching experiments are introduced which were developed to change and reform current teaching practice. It is suggested that such a design holds much promise for instituting significant positive changes by establishing a better connection between research and practice. (KHR)

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Transformative Teaching Experiments Through Conjecture-Driven Research Design

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Educational research must serve a variety of purposes. One of these purposes is to invent, develop, and test novel ways of teaching mathematics. Until quite recently, research frequently was confined to out-of-school laboratory settings (Kilpatrick, 1992). This allowed theories to be tested in a fairly controlled environment. However, results and findings from these types of studies have been criticized for being of limited applicability to classroom instruction (Romberg & T. P. Carpenter, 1986). The basis for this criticism was that the instructional methods used in these experiments were not subjected to the constraints of schooling, and, thus, teachers were skeptical that they would prove sustainable in everyday situations.

Such critiques led to a period in which we witnessed an increase in research on existing exemplary classroom practices and an argument that new practices need to evolve from the "best practices" that exist in schools currently. However, some researchers worry that the development of best practices for wide-scale implementation would require a significant period of study and theory building. This might result in a reform movement that is too constrained, too incremental, or too delayed to meet the needs of the country's children. Therefore, there is a need for classroom research that is more speculative and in which some of the constraints of typical classrooms are relaxed while others remain in force.

In this chapter, we describe a research design model that utilizes both theory and common, core, classroom conditions in order to create and investigate new instructional strategies. The strategies developed through this design are meant to change and even to reform current

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teaching practices drastically. We call this design *transformative and conjecture-driven teaching experiments* and we argue that such a design holds much promise for instituting significant positive changes by establishing a better connection between research and practice.

DEVELOPING THE CONJECTURE

Ideological Stance

We want to make clear that we situate this design in the realm of social science. More and more, social scientists are recognizing how our values and beliefs frame our choice of research problems, our selection of methods, and our interpretation of data (Guba & Lincoln, 1989; J. K. Smith, 1989). Take, for example, the traditional emphasis on computational skills in mathematics assessments. Such an emphasis grew out of our society's value of computational skills (National Council of Teachers of Mathematics, 1989). These skills allowed students to secure jobs and to become informed citizens in an industrial society. However, with advances in technology, such computational skills are no longer as important. Instead, students need to develop critical-thinking skills to interpret data appropriately and to use technology to solve more complex problems. Thus, changes in our society have led to a change in what we value in mathematical skills. With this change in values has come a change in the mathematics education community's research agenda.

In any aspect of social science, there are many examples like the one aforementioned where it is clear that what is valued by society and/or various research communities guides the specifics of a research agenda. Furthermore, because our particular field of social science is education, not only will interpretations derive from this work, but also the research itself inevitably will constitute an intervention. We believe that because education consists of ongoing practices, intervening through research makes "replication," as a simplistic concept, unattainable. However, we claim that the ways in which research acts as an intervention will inform discussions on how the research is carried into practice.

Thus, we are obligated to acknowledge the influence of our values on our interpretations, while ensuring that our interventions are educationally sound and reflect our values. In recognition of this, we make our values explicit before discussing the specifics of our research design model. We unapologetically acknowledge that our experiences, beliefs, and values influenced the shape and structure of our research model.

Our transformative and conjecture-driven teaching experiment is motivated by our commitment to equity, and we consider this the basis of our ideological stance.¹ In our work, we seek continually to advance the kind of research that entails active attempts to ensure equal opportunities for all students to participate in and succeed at mathematics. There are several reasons for this stance. The first one stems from current problems with the lack of equity in mathematics education.

Among students in levels as low as middle school, researchers have found significant gaps in achievement in mathematics between different groups of students (girls vs. boys, African American vs. White, lower socioeconomic groups vs. middle and higher socioeconomic groups, etc.), (Secada, 1992). These achievement gaps translate into participation gaps in high school and college where researchers have documented a decline in the numbers of ethnic and racial minorities participating in higher level mathematics courses and an underrepresentation of women in quantitative fields. Nonparticipation and a high rate of failure in mathematics by underrepresented groups mean limited access to certain, higher paying prestigious careers and jobs and a lack of influence on the course of technological development (Secada, 1992). Even among the entire population, we see inordinate levels of failure in mathematics. As it exists, we are supporting an elaborate mathematics education system that succeeds for only a tiny percentage of the population.

Because we see these inequities as stemming from how mathematics education is structured, we see unfairness and unnecessary failure arising from a myriad of common instructional methods. Typically, mathematics is taught with an undue and a premature emphasis on the abstract and formal, which, we argue, prevents many students from accessing mathematical concepts. In many schools across the nation, students are assigned to mathematics courses by the use of "ability grouping" schemes that are often arbitrary and lack flexibility and bidirectionality (Oakes, 1990). This lack of equal access to opportunities to learn, to resources, and to sound instruction leads to the systematic disenfranchisement of certain groups (Oakes, 1990; Secada, 1992; Tate, 1994).

Finally, we affirm that our stance, formed by our belief that mathematics is a human construction, would benefit from the inclusion of contributions from more diverse constituencies. We hold this commitment in spite of a well-documented history of

¹Because ideological stance, unlike political correctness, is not merely an attitude that one assumes or discards, we are not prepared to consider whether other ideological stances fit with this methodology, but we would not rule out such considerations by others.

mathematics that suppresses and excludes broad participation and point instead to other fields in which struggles to overcome bias have succeeded in enriching the quality of the scholarship (e.g., the disciplines in the humanities that are influenced by women's, Afrocentric, Latino, and Asian studies).

In mathematics education, we propose to incorporate diversity by close listening. Years of experience of working with children and listening to their ideas and innovations carefully demonstrate that they frequently produce insights and innovations that have escaped the experts. The variety of questions, representations, and solutions that students have constructed has both intrigued and astounded us. However, recognizing the novelty and power of students' ideas takes practice, an expectation of diversity in approach, and a willingness to envision things from their point of view (Confrey, 1991, 1993, 1994).

Consequently, into our ideological stance, we place the belief that a novice can often envision possibilities, arrangements, and logical relations that experts have been trained to overlook. Successful close listening requires the expert to revise his or her perspective in light of a student voice as well as to acknowledge the source of these revisions in the teaching and learning acts. This can be particularly difficult given the dominant culture's portrayal of mathematical constructions as pure, highly symbolic, and existing outside human experience. This makes one unlikely to hear students' constructions and, therefore, limits and narrows them. Furthermore, classroom climates that emphasize only the quick and ready display of mathematical competence lead to the suppression and destruction of "voice"; students learn to stifle and deny their insights, creating oppressive circumstances. The researcher must be able and willing to allow these novel constructions to emerge and support them, despite the limitations potentially set by the content under examination, the classroom environment, and mathematical tradition (Confrey, 1991, 1994). We describe the examination and revision of expert perspective in light of student voice as a *voice-perspective heuristic* and use it to guide our interventions and interpretations.

Our claim is that one's ideological stance informs and saturates one's research design model. Having defined our ideological stance, we now describe the components of our design model.

The Conjecture

A crucial part of this type of teaching experiment is the conjecture. But what is a conjecture? According to the dictionary, one definition of a conjecture is an inference based on

inconclusive or incomplete evidence. In the context of mathematics education, this inference may pertain to how mathematics for educational purposes should be organized or conceptualized or taught. Thus, the conjecture is a means to reconceptualize the ways in which to approach both the content and the pedagogy of a set of mathematical topics. Most often, it comes from a dissatisfaction in the researchers' minds with the outcomes of typical practices. It transforms how one views teaching and learning activities. Over the course of the teaching experiment, a strong conjecture should shift one's perspective and bring new events, previously insignificant or perplexing, into relief. At points in its evolution, the conjecture should feel like a grand scheme beginning to emerge from many, previously disparate pieces, making them more cohesive.

Unlike a formal hypothesis in an experimental design approach, a conjecture is *not* an assertion waiting to be proved or disproved. Research guided by a hypothesis merely attempts to discover if a given intervention worked or not or if a given theory was supported or not. But research guided by a conjecture seeks to revise and elaborate the conjecture while the research is in progress. Thus, whereas a hypothesis remains static throughout an experiment, a conjecture evolves constantly as the research progresses. This is similar to Lakatos' (1976) recognition that theory is established "through the incessant improvement of guesses by speculation and criticism" (p. 5).

The conjectures that we envision as essential in this type of research have two significant dimensions. First, the conjecture must have a mathematical content dimension. This means, for example, that a conjecture would not concern itself with the impact of conflict management skills on group problem solving. Instead, the conjecture would have to include a mathematical component such as how a full understanding of radical mathematics is critical to low-socioeconomic-class students gaining full citizenship in a technological society (Frankenstein, 1995) or how the mathematics of change can lead to the introduction of calculus-related concepts as early as elementary school (Nemirovsky, Tierney, & Wright, 1995; Roschelle & Kaput, 1996b; P. W. Thompson, 1994a). Although the content of the conjecture does not have to come out of traditional school mathematics curricula, the researcher should be able to define and describe clearly the mathematical content of the conjecture. In brief, the content dimension answers the question: What should be taught?

Second, there must be a pedagogical dimension linked to the content dimension that answers the question: How should this content be taught? This aspect of the conjecture guides the researcher on how the classroom needs to be organized for instruction and on what kinds of tasks, activities, and resources need to be provided for the content.

For example, in the Algebra Project (Silva, Moses, Rivers, & P. Johnson, 1990), Moses recognized the role that algebra serves as a critical gateway to success and participation in higher level mathematics for inner-city students. He conjectured that what is needed to help students move successfully from elementary school arithmetic thinking to high school algebraic thinking is a transitional curriculum. At the same time, Moses claimed that students must develop these transitional skills through familiar physical experiences and must be supported in their learning by an organized network of community members. Consequently, the content of Moses' conjecture is the need for a curriculum that will teach transitional skills that allow students to access to abstract algebraic notation. The pedagogical dimension of the conjecture becomes the teaching of these skills through engaging and experiential tasks with explicit and intensive community involvement and advocacy. This conjecture crystalizes the notion that as the level of symbolic abstraction is raised, students need to work with experientially familiar frames of reference that help them to learn algebraic notation.

The Theoretical Framework

A conjecture is necessarily situated in a theory or it cannot be interpreted. Theory, such as intellectual development theory, social organizational theory, or cognitive theory, relates the conjecture to other aspects of education or mathematics. Whatever the particular theoretical aspect of the conjecture is, it serves to structure the activities and methodologies in the teaching experiment. In relation to the conjecture, the theory helps to weave together the content and pedagogical dimensions.

A critical element of the theoretical framework is a careful review and analysis of the existing literature. Seldom, if ever, does a robust conjecture fall full-blown from the sky. Usually it comes from a thoughtful and critical review of the literature to discern an anomaly that has been overlooked, unsolved, or addressed inadequately by one's colleagues. As a result, a theoretical discussion should funnel one from a broad set of theoretical claims to the specifics of the content to be imparted and how the conjecture is situated in that literature. A full discussion of theory is beyond the scope of this chapter, but, in the next section, we present a brief discussion of the theories that guided the "splitting" conjecture. Its role in guiding the research, in defining what counts as evidence, and in creating the observational categories is well documented in other literature (Steffe & Gale, 1995).

An Example: The "Splitting" Conjecture and Related Theory

To illustrate better what we mean by a conjecture, we describe an example from our work involving teaching experiments. In this case, the content of the conjecture was a topic known as *splitting*. The notion of splitting (Confrey, 1988, 1989, 1994; Confrey & E. Smith, 1995) grew out of Confrey's research into students' conceptions of exponential functions. From this work, Confrey and her research team began to see that mathematics curricula tend to rely too heavily and almost exclusively on counting and, consequently, on the model of multiplication as repeated addition. According to Confrey (1994), the problem with conceptualizing multiplication exclusively as repeated addition is that such a conception cannot accommodate many of the students' intuitive and primitive actions that *are not* repeated addition but that often *are* multiplicative in nature. Some of these actions include sharing, folding, dividing symmetrically, and magnifying.

These types of action, Confrey (1994) claimed, are the basis for a "primitive, cognitive scheme" that she called splitting: "In its most primitive form, splitting can be defined as an action of creating simultaneously multiple versions of an original, an action often represented by a tree diagram" (p. 292). In this sense, students are splitting when they divide a piece of paper into four equivalent sections or break a candy bar in half and in half again to share it among four people. They are splitting also when they are organizing a set of objects into an array because it requires the production of groups of equal size. When students do this, they are not relying on counting to verify that they split the areas or sets correctly. Rather, they are relying on the geometric equivalence of the resulting parts or a one-to-one (or a one-to-many) correspondence of the parts. Thus, the splitting construct can live independently of counting and repeated addition.

The construct of splitting provides a foundation and a context for the concept of ratio (Confrey & E. Smith, 1995). One of the attributes of ratio that emerges from the splitting conjecture is that it is connected inherently to the operations of multiplication and division. Through splitting actions, multiplication, division, and ratio form a trio of mathematical ideas that mutually support and define each other. Confrey (1988, 1989, 1994) argued that in order to support the intuitive splitting actions of children, students should be introduced to the constructs of multiplication, division, and ratio as a trio of ideas while they are in the primary grades. As a result, the content component of the splitting conjecture proposes the transformation of traditional school mathematics curricula so that, in addition to exposing

students to additive structures, it also will support their understandings of multiplicative structures.

The literature on multiplication and division and on ratio and proportion was critical to the evolution of this claim. The Rational Number Project (Lesh et al., 1989) had identified already myriad of rational number constructs that begged to be simplified and unified. The work of Fischbien, Deri, Nello, and Marino (1985) on primitive notions had an asymmetry; division had two roots (partitive and quotative) but multiplication had only one—repeated addition. Alleviating this asymmetry was a powerful impetus in the proposal of the conjecture. The careful work of Steffe and colleagues (Steffe, 1994) on number made counting the centerpiece and eventual underlying structure of multiplication, and yet counting failed to capture the geometric intuition of arrays, symmetry, and similarity, which also are multiplicative in nature. And, finally, the conceptual change literature (Strike & Posner, 1982) prompted a historical investigation that revealed that the Greek concept of ratio was lost in the modern arithmetization of number. Each of these literature reviews pushed Confrey to articulate a new conjecture.

For the authors of this chapter, the pedagogy of the conjecture was embedded deeply in our theoretical framework, which we would describe as situated in a constructivist paradigm informed increasingly by a sociocultural perspective and a sensitivity to students' voices (Confrey, 1994, 1995, 1995). In mathematics education, constructivist theory makes the claim that all mathematical ideas are grounded in human schemes of action. Mathematics is viewed as a result of personal constructions made through one's actions and reflections on those actions. In a teaching situation, mathematical constructions are influenced by the conditions that spur children to action as well as the experiences and insights that children bring to those actions (Confrey, 1994, 1995, 1995; von Glasersfeld, 1984).

At the same time, all actions and reflections are assumed to be saturated by one's cultural settings and interactions. Typically, actions do not take place in a vacuum, but involve social interactions with other people. Usually, the norms of the environment in which these interactions take place are dictated by the dominant culture. Thus, students' mathematical constructions are influenced by the social and cultural norms that regulate their activity. In this perspective, language serves as a means of both acculturation, personal expression, and exchange (Confrey, 1994, 1995, 1995; Vygotsky, 1986).

Our ideological stance influenced the conjecture in both of its dimensions of content and pedagogy. Ratio and proportion are critical topics for success in advanced courses, and, as a

result, increasing the success of a diverse group of students' with them is a worthy target for reform. That increased success would come with the support and articulation of student voice was a central component of the research. In this manner, we see how theory and ideology interact with the construction and articulation of the conjecture.

In addition, theory and ideology affect all of the components of instruction designed to operationalize the conjecture in actuality. In our case, this interaction resulted in the intervention's attention to the following areas:

- In order to construct knowledge, students need opportunities to explore activities that are rich in possibilities for multiple solutions and various tool use.
- Students need tools, materials, and other people with which and with whom they can build the constructions associated with these activities.
- As new starts and alternative approaches surface, structures to support and develop them must be present.
- The listener or researcher must be compelled to examine simultaneously and dialectically both his or her own understanding of the ideas under construction (and their places in sociocultural history) and understandings of the students.

DEVELOPING THE TEACHING EXPERIMENT²

Having defined and described what we mean by conjecture, we outline now what a transformative and conjecture-driven teaching experiment is. We see the teaching experiment as a planned intervention that takes place over a significant period of time in a classroom where a continuing course of instruction is taught. The experiment will involve a dialectical relationship between the conjecture and the components of instruction, which are: (a) the curriculum; (b) the method of instruction; (c) the role of the teacher; and (d) the methods of assessment.

² We have chosen to retain the label *teaching experiment* despite the connotations of the term *experiment*. This is because the term has established itself with a varied set of meanings in mathematics education, as demonstrated by its different uses in chapters in this book (e.g., Steffe & Thompson, chap. 11, this volume).

The research questions and the methods of data collection and analysis must be informed by both parts of the dialectic, and it is in this redefined relationship that we see the basis for the redefinition of the research-practice relationship. We offer a general description of each of the components of instruction and illustrate how we developed those components in our work.

It should be noted that this form of complex research requires a team of researchers working on the project. This is not merely a response to the need for multiple data collectors, rather because of the need to discuss and refine our interpretations and plans continuously. Identifying the appropriate roles of the team's members and their foci of attention are important parts of the project. We found it useful to have individuals who would concentrate on the design and analysis of separate segments of the curriculum, whereas the project director was continuously responsible to guide the overall directions and work for the elaboration of the conjecture. Within a university where graduate students are in different stages of their studies, this design works very well.

Refining the Conjecture and Developing Research Questions

As stated previously, the conjecture is the driving force in this research design model. It will determine exactly what questions the transformative teaching experiment will explore. Because so much of the research design is dependent on the conjecture, it might be wise to do some preliminary smaller investigations into the power of the conjecture before beginning a large-scale teaching experiment based on it. These smaller investigations might involve working with a small group of students for a short time to explore a single aspect of the conjecture. This will lead to a better understanding and definition of it. Ultimately, such a process will frame the questions for a full-scale, conjecture-driven teaching experiment.

In the case of the splitting conjecture, several smaller studies had contributed to the evolving construct of splitting. One study focused on college students' conceptions of exponential functions (Confrey, 1991), which led to the understanding of how students reason with multiplicative units. Another study looked at first-grade students' understanding of similarity (Confrey, 1992) and found that similarity is a strong intuitive notion for young children. The results of these early investigations of splitting confirmed, among other things, that students do have a variety of conceptions of multiplication that go beyond repeated addition. Furthermore, as mentioned earlier, an examination of history revealed that the Greek conception of ratio was very different from the construct held today. The Greeks' idea of ratio was grounded in action just as students' early experiences with splitting are believed to be

(Confrey & E. Smith, 1989). Through these various activities, the construct of splitting was refined and strengthened. Eventually, this process led to these questions:

- How would mathematics education be different if the intuitive notions of splitting were supported?
- What would curricular activities that support splitting look like?
- What mathematical strengths and weaknesses would students develop if their sense of splitting were developed?

These questions led to a more extensive teaching experiment using the splitting conjecture.

Because components of this teaching experiment are discussed throughout the rest of this chapter, a brief overview of the experiment is provided now. Over a 3-year period (1992–1995), Confrey conducted a transformative and conjecture-driven teaching experiment with a heterogeneously grouped class of elementary school students ($N = 20$) in a public school of a small city. Confrey taught most of the sessions. A research team consisting of two to four graduate students (depending on the year) assisted Confrey.

During the course of the experiment, the elementary school students passed through Grades 3, 4, and 5. In the third grade, the students were introduced to multiplication, division, and ratio as a trio of mathematical ideas. When this same group of students was in the fourth grade, their instruction focused on strengthening the construct of ratio and introducing fractions as a subset of ratio (Confrey & Scarano, 1995). While studying ratios and fractions, the students worked in such contexts as scaling recipes, using shadows to estimate height, and building an access ramp with an appropriate slope for handicapped persons. As fifth-graders, after having developed a rich network of mathematical ideas, the students were introduced to decimal numbers. Further details of this teaching experiment can be found in Confrey (1995, 1996), Lachance (1996), Preyer (1996), and Scarano and Confrey (1996).

In instruction, we identified four design and implementation components that need to be conceived and carried out progressively during the teaching experiment. These are: (a) curriculum; (b) classroom interactions; (c) teaching; and (d) assessment.

Four Design Components of Instruction

Curriculum. The conjecture will determine what content areas will be covered by the curriculum used in the classroom intervention. However, because the teaching experiments take place in classrooms where a course of instruction proceeds continuously, the researcher must take into account what students and teachers are expected to cover at that grade level. Most schools must answer to a system of accountability—such as end-of-year school district

examinations or national standardized tests—to ensure that the designated material has been covered.

There are many ways to handle this. The researcher can plan to conduct the teaching experiment during a period of review so that the teacher and students will have ample time to cover the prescribed material. Or the researcher could propose to cover the content dictated by the school district's mandate within the confines of the teaching experiment. Or the researcher could get a waiver for the teacher and students involved in the teaching experiment so that they will not be held accountable for the material that could not be covered because of the time used by the teaching experiment.

Whatever the researcher decides to do, it needs to be done in consultation with the teacher and the other relevant parties (parents, administrator, and others). The setting of this type of research in working classrooms brings with it an ethical responsibility to ensure the progress of all of the students. This means that one must accept the commitment to the students, the parents, and the staff members to act only on behalf of the children's academic and emotional well-being. Should this commitment require one to compromise the quality of the research, the commitment must prevail. Typically, what happens if an accommodation has to be made is a modification of the curriculum that is a compromise between what the researcher might wish to do and the practical demands of the setting.

An example of a compromise occurred in the splitting research when we needed to teach the addition of fractions. Our dilemma was that in the two-dimensional plane, ratios, as vectors, can be added with vector addition. However, conventional mathematics frowns on notating this as:

$$\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$$

although combining ratios is a somewhat commonly encountered operation. Because we were concerned that our students not be penalized on the end-of-the-year achievement tests for using this type of operation in an inappropriate context, we gave them a project involving the addition of measurements followed with an assessment asking them to evaluate the responses of two students. In the assessment activity, the response of one student was

$$\frac{2}{5} + \frac{2}{5} = \frac{4}{10}$$

whereas the other was

$$\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$$

In our classroom discussion, we made it clear that only the second of these responses would be scored as correct on typical achievement tests, even though the first notation made sense in certain contexts.

The major influence on the content of the teaching experiment's curriculum is the conjecture and determining how it could influence the choice, sequence, and duration of what is presented as well as the types of activities used. In discussing the curriculum component of a the teaching experiment, it is essential to point out that one cannot design a complete curriculum in advance. If it is truly a conjecture-driven intervention, rather than a program implementation, one enters it knowing that future decisions in the experiment will be made based on what happens during the earlier parts of the intervention. We have called this type of curriculum development "responsive and emergent" (Lachance, 1996).

Although the researcher certainly will have specific content areas and topics that he or she intends the conjecture-driven intervention to cover, curricular activities should be structured so that they are flexible and open-ended. This enables the researcher to observe and monitor students' interactions with the curriculum and its activities. As a result of these observations, the researcher can adapt curricular activities according to the responses and innovations contributed by the students. Hence, the development of this type of curricular intervention can be seen as responsive.

Responding to students' needs and inputs also makes the curriculum development process in this type of research design emergent. As the students work through the curriculum and offer different and often innovative insights into the mathematics, the researcher's own insights into the content will evolve. This is expected to be particularly striking because the researcher is looking at the students' work through the lens of the conjecture.

If the conjecture is not robust, there will be insufficient information to start and then guide the new design. On the other hand, if the curricular intervention is not aggressive enough, the guardians of the traditional approach may resist and drown out the new intervention eventually. Moreover, if the curriculum and the conjecture are both robust enough, the changes in the curriculum should be more substantial than those in the conjecture. As we discuss further in the section on the preliminary data analyses, although changes in the conjecture are expected, these changes can be described as "evolutionary

refinements" and "elaborations." The conjecture's changes are in contrast to the curricular changes that are responsive and emergent. Relatively, the rate of modification of the conjecture should be slower than that of the curriculum. An analogy might help. Imagine looking at a complex and changing scene through a lens. As the scene changes dramatically, small adjustments with the lens must be made in order to bring all of the aspects of the scene into focus. In relation to the changes in the scene, the focus adjustment is incremental. Because the conjecture functions as a lens to the complex scene, we expect small shifts in its position, but a deeper elaboration of it when viewing the sometimes major "scene" changes of the curriculum.

In the splitting conjecture experiment, both the splitting content and the constructivist theory influenced the types of curricular activities used. In terms of content, it was the intention of our curricular intervention to cover many of the topics mandated by the traditional elementary school mathematics curricular for Grades 3 through 5, such as multiplication, division, fractions, decimals, and geometry. In addition, we sought to expand the content of the traditional curricular by deepening the treatment of the traditional topics and introducing new arrangements such as making ratio and proportion by two-dimensional graphing superordinate to fractions and including an early introduction to algebra.

To accomplish all of this, we developed a model for approaching these topics through the splitting conjecture. This model (see FIG. 10.1) has at its center the trio of mathematical constructs of splitting: multiplication, division, and ratio. It is through this trio that other topics such as fractions, decimals, geometry, two-dimensional graphing, and, eventually, percents can be developed and explored.

With our intervention's use of a wide range of concrete and contextual experiences, we hoped to encourage students to develop a firm grounding in the central trio of constructs. From this foundation, richer understandings of all related subsidiary constructs could be developed and the conjecture could be elaborated. Because this model views the subsidiary constructs as being connected to and through the central trio of concepts, the relationships between and among all of the concepts can be explored, seen, and understood more readily by students.

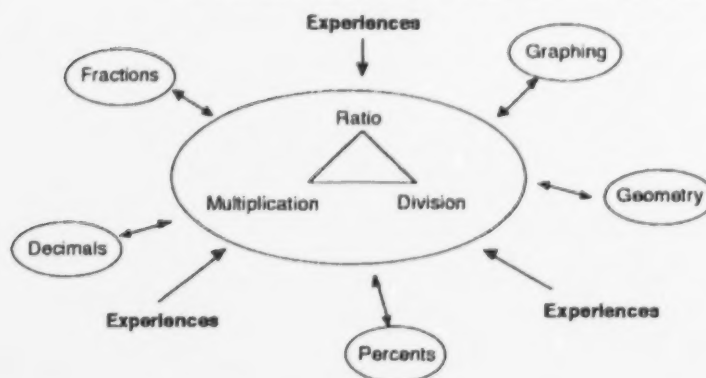


FIG. 10.1. A model for approaching selected topics through a trio of splitting constructs.

Consistent with our theory, the curriculum was organized around modeling activities where students were engaged in tool-based investigations. Although the activities in the splitting experiment were called for at different points throughout the intervention, they did have a number of common characteristics. They were designed to be fun and compelling, with the intent of creating the need for students to construct various understandings. Consistent with our equity stance, the activities included experiments, demonstrations, puzzles, and design projects, one of whose purposes was to make it easy for all of the students to begin them. Because many of the problems had multiple facets with varying levels of challenge, all of the children in the class learned that they could participate in these projects successfully and advance to the next activity. Similarly, these tasks typically allowed for differing degrees of completion, permitting interested students to pursue them further. Finally, with a strong use of context, commonplace tools, and varied student experiences the problems were widely attractive and generally offered multiple possibilities for their representation (i.e., tables, graphs, pictures, etc.). All of this was essential to create sufficient space for interesting and novel approaches to emerge.

For instance, in Grade 4, using their understanding of ratio, the students had to design a wheelchair ramp that would fit a specific entryway and have an appropriate slope and supports. Eventually, actual ramps were constructed with each group's chosen slope. To test their designs, the students traveled up and down the ramps in a wheelchair.

The conjecture provides the impetus to develop careful intra-activity sequencing as well as interactivity sequencing to allow the systematic, albeit emergent, conceptual development of mathematical ideas. For instance, the curriculum activity of the handicapped ramp problem emerged as we elaborated the conjecture. We knew that we wanted to use the two-dimensional

plane to compare ratios and to progress into operations with them. We entered the teaching experiment with a conjecture that splitting had strong geometric connections, and we sought to bring them out in our curricular examples. Constructing a ramp met our needs on a variety of levels. The students could engage in tool-based activities, they could work in a context with which they are familiar (especially in schools as mainstreaming is implemented), and they would make the concept of slope an effective bridge from ratio and proportion to algebra and geometry.

As we approached the time to undertake the ramp problem, we realized that the students needed more experience in working with large whole numbers and a fixed ratio relation. This led us to insert an activity that involved finding the heights of trees from the lengths of their shadows. Again, one can see how curricular changes are more likely to be revised in response to classroom observations, whereas the implied changes to the conjecture are more delicate and refined.

Classroom Interactions. Because this type of research necessitates a teaching experiment, the researcher will need to decide how the instruction will be structured. Will the students work individually, in small groups or as an entire class? Should there be problem solving activities or a lecture format? Or will there be a combination of these instructional approaches? Both the content and the pedagogical dimensions of the conjecture, informed by the theoretical framework, will influence the methods of instruction chosen. In addition, the types of curricular activities that the researcher hopes to employ will dictate in some respects the forms of classroom interactions used in the intervention. Decisions on the forms of these interactions are essential in planning the methods of data collection.

Because of the emphasis on the use of tools and model-building tasks, much of the activity in the splitting experiment took place in small heterogeneous groups of students. The groups remained intact during particular activities, with changes in participants occurring as new activities were introduced. Before, during, and after small-group work, sessions were devoted to whole-class discussion. Sometimes, these whole-class discussions were used to "create the need" for the activity or to share and discuss progress during group work on the activity.

However, because of the conjecture's focus on the students' voices, the whole-class discussions also allowed for the development and emergence of students' voices in relation to the teacher's and the researcher's perspectives. In these exchanges, it was the interactions

among the students and between the teacher and the students that determined what would be discussed and pursued, and ultimately, what meanings would be negotiated.

The Role of the Teacher. In a transformative and conjecture-driven teaching experiment, two questions related to the role of the teacher must be answered. The first is: Who will act as the teacher during the intervention? In some situations, it may make sense for the current classroom teacher to be the main teacher in the intervention. This would allow the researchers to spend more time observing the interactions taking place in the classroom and the effects of the intervention. However, in order for the classroom teacher to be the teacher during the experiment, she or he would have to agree to be a full member of the research team. This means that the classroom teacher would have to be very familiar with the conjecture. It also implies that the teacher would have to be able to participate in the intervention's development and to assist in analyzing the data from the experiment during both the preliminary and final analyses. Obviously, this would entail a substantial commitment of time and energy by the teacher. Because of these requirements, it may be better to have a member of the research team act as the classroom teacher during the intervention. Clearly, the particular characteristics of the classroom setting and the people involved in the research will influence how the teacher for the teaching experiment will be chosen.

No matter who acts as the teacher during the experiment, a second question needs to be answered: What role will the teacher play during the intervention? Will the teacher act as the expert, dispensing knowledge as necessary? Will the teacher act as the facilitator, assisting students as they act on their own or in small groups? Or will the role of the teacher change according to the curricular activities? The answers to these questions must be informed by the conjecture itself and by the types of curricular activities that the intervention will use.

In the splitting experiment, the leader of the research team acted as the classroom teacher. The novelty of the splitting conjecture and its lack of extensive articulation at that time made it both too difficult to communicate to a classroom teacher as well as too onerous, risky, and threatening to the fidelity of the study to have it taught by someone insufficiently versed in it.

As the teacher, Confrey took on a variety of roles. At points, she was an expert resource person, a listener, a facilitator, a discussion leader, a coach, and an evaluator. As both a teacher and a researcher and given the focus of the splitting conjecture, she had to approach the task of teaching with a profound respect for and curiosity in the possible alternative approaches used by the students. She also had to draw upon a broad knowledge of the content, which sometimes meant that she needed to confer with mathematical experts on such topics as vector spaces, linear algebra, and projections. This was true despite the fact that the intervention was

at the elementary grade level. Furthermore, because one of the tasks of the research was to discern the students' novel approaches to the conjecture and to assess their fruitfulness, listening was one of the teacher's major responsibilities.

Assessment. As with the other components of the teaching experiment, the assessment activities used to evaluate the students' learning and progress must be informed by and consistent with the content, pedagogy, and theoretical framework of the conjecture along with the other components of the intervention. The results of assessments not only will provide outcome data related to the impact of the intervention, but also will allow for an ongoing formative evaluation of the intervention process. This formative assessment will guide the evolution of the conjecture and the related curricular materials continuously. Thus, it will be important for the researcher to vary the types of assessment practices used and to employ both informal and formal evaluations.

In the splitting experiment, multiple assessment practices were employed. For example, group projects always ended with a performance assessment. Often, a small panel of guest evaluators were invited to the classroom so that students could present and defend their solutions to problems. As the intervention continued and group work progressed, the students themselves often became the best source of questions about and criticisms of other students' solutions.

Homework was assigned four nights a week and was collected and corrected. This gave the research team an opportunity to perform frequent checks of the students' understanding. Short, in-class assignments, which the students might work on individually or in groups, also were used to gauge the group's general understanding of the topic.

Other assessments were used to evaluate the overall impact of the intervention. The researchers created longer, end-of-unit assessments that consisted of open-ended tasks or nonroutine problems suggested by other research. The regular classroom teacher also administered tests on skill development and gave the researchers access to their results. Finally, standardized achievement tests, mandated by the school district, were taken by the students. The district made these results available to the researchers in order to ascertain how the intervention might have influenced students' performance.

Putting It All Together: The Research Design Model

We can now present a model for the transformative and conjecture-driven teaching experiment (see FIG. 10.2). The teaching experiment is embedded in social science and education, an

ideological stance on equity, diversity, and the importance of student voice and a theoretical framework. The conjecture and the instructional components are drawn as separate entities in a dialectical relationship. Together, they are labeled the teaching experiment. The two dimensions of the conjecture and the four components of instruction are listed.

The conjecture, the instruction, and their relationship change over time, which is signaled by their dialectical relationship and by the two feedback loops. The feedback loop to instruction is labeled *emergent and responsive*; the feedback loop to the conjecture is labeled *evolving and elaborating*.

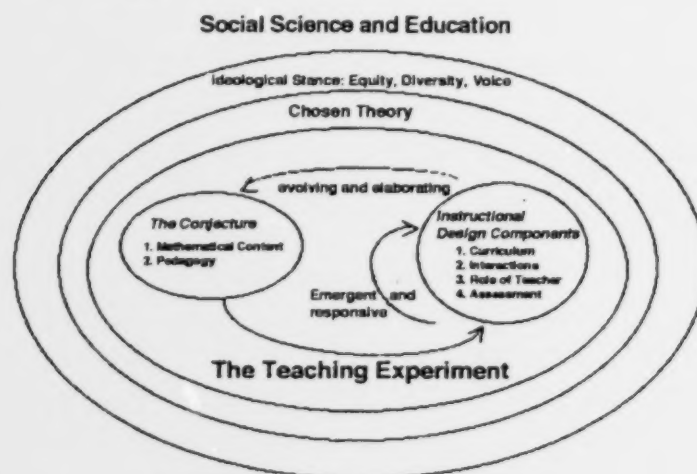


FIG. 10.2. A model for the transformative and conjecture-driven teaching experiment.

DATA COLLECTION AND ANALYSES

Data Collection

In a transformative and conjecture-driven teaching experiment, multiple methods of collecting data should be used. First of all, there should be some way to capture the day-to-day interactions in the classroom. One of the most straightforward methods of doing this is videotaping classes. A combination of videotaping and careful notes of classroom observations is often ideal.

In addition, some data should be collected on individuals. Certainly, some of the assessment activities provide data on what students have learned. However, more in-depth understanding of students' learning and development can be obtained through task or open-

ended interviews. Again, the content and the theoretical framework of the conjecture will help to determine both the number and the form of the interviews.

Following a group is also a valuable source of information in classes where children are taught by the group instructional method. In small groups, the articulation of students' voices is rich and revealing of their conceptions. The decision to follow one group instead of multiple groups is a difficult one. The trade-off is to witness the conceptual development of one group more fully as opposed to broadening one's sampling of students' methods by viewing multiple groups.

Finally, and importantly to note, data need to be collected about the researchers' thinking about and actions during the intervention. Because so much of this type of teaching experiment depends on what emerges from the classroom interactions, such data are vital to refining and understanding the conjecture and its impact. Ideally, video and/or audio records of the researchers' meetings should be made. Furthermore, the researchers should keep field notes or journals to record their thoughts and reflections about the intervention as it continues. It even may be helpful or necessary for a member of the research team, or perhaps someone outside the research team, to interview the different researchers about the research process.

In the splitting experiment, videotaping was the primary form of data collection (see Roschelle, chap. 24, this volume). Both the whole-class and small-group interactions were taped. Generally, two cameras were used. During whole-class activities, one camera recorded the presenter at the front of the room, who could be the teacher or one of the students, along with the board or large paper display. The second camera recorded the students and tended to focus on the student who was speaking.

During small-group work, a study group was chosen and one camera followed it throughout a series of activities in a given unit. The second camera moved among the remaining groups to record the diversity of methods that emerged from the students' work. Using this method, we were able to gather data that allowed us to follow the conceptual development of one group, while collecting a sample of data from the other groups.

One technique used to achieve better insight into what the students were thinking and doing was to have small groups give a progress report to the entire class. Sometimes, these progress reports took the form of the groups recounting their work from the previous day. At other times, the groups discussed their current plan of action. On occasions, the groups shared their concerns about the obstacles they faced in accomplishing a task. These reports were a source of significant data.

Besides the videotapes, other sources of information included the students' written homework, and other written assessment activities, and the verbal responses gathered from them during intermittent interviews. As mentioned previously, homework was given, collected, and corrected each week in order to maintain a continuous record of students' conceptual development. Also, low-stakes assessments involving short (less than 15 minutes), open-ended items were used to gauge the depth and breadth of the students' understanding.

Generally, we interviewed students when we were unsure of their perspectives on and depth of understanding of a certain topic. Some of these interviews took place as a student worked on the short, low-stakes assessment items described previously. Additionally, at the end of the decimal unit, the members of the study group were interviewed to ascertain how they had made sense of the entire decimal unit.

The records of our own meetings and thinking tended to be kept by a member of the research team. For the most part, these records were summaries of the contents of a given discussion as opposed to verbatim transcripts. Usually, such meetings were not video- or audiotaped because they took place mainly in the classroom or in the car on the drive back from school directly after the class session. Given these circumstances, keeping minutes of these conversations seemed to be a more effective method of recording their content. Some members of the research team also kept field journals to track their thinking during the intervention.

Preliminary Data Analysis and Curricular Revision

In a transformative and conjecture-driven teaching experiment, we see two types of data analysis. The first type is the ongoing and preliminary analysis, which takes place during the intervention. This is necessitated by this design's anticipation of emerging issues. Thus, at frequent intervals (perhaps after each class), the research team needs to meet and discuss the implications of the students' activities for future plans. Although this extensive reflection on practices and careful modification of plans could not occur to the same extent with a regular teaching load, it is essential for the success of the teaching experiments. It is through this preliminary analysis that both the conjecture and the curricular activities are reflected upon, developed, and revised.

In the splitting experiment, there were several objects of preliminary analysis. Generally, almost no stable analysis could take place in the first few days of a new activity. Typically, these were chaotic and noisy times. The intent of the projects would be clarified, and the

research team would feel overwhelmed by the level and variety of activity. Seldom did things go as planned. During this time, it was difficult to conduct any kind of systematic examination of the data, although the research team met nearly every day, if only to recount the day's tumultuous events.

However, by the end of two or three days, coherence would emerge. Students would have transformed the activity into their own, would have asked questions to clarify the task, and would have made a reasonable schedule and timeline for completing it. Simultaneously, the research team became more confident of their observations and ability to respond to the emerging needs of the students.

At points in every activity, the teacher-researcher would hold whole-class discussions with a specific goal in mind. The topics of such discussions could be a review of a specific homework problem that might inform a current activity or the teaching of a particular skill that students saw necessary for the larger, problem solving activity. These discussions were important territory for an initial analysis of the data. The students' proposals, how they represented the problem (tables, graphs, pictures, etc.), and their mode of language were examined carefully.

For instance, as reported elsewhere (Confrey, 1995), in one group discussion, the students were asked to find different amounts of lemon juice and water that would taste the same as a recipe for lemonade that consisted of 9 parts lemon juice to 12 parts of water. They did this by searching for the smallest whole number ratio (3:4) that fit perfectly into the given ratio (9:12). When asked what this could be called, they replied "the littlest recipe" or "the base combination." The first of these two terms ("the littlest recipe") was used extensively throughout the teaching experiment by the students to describe a ratio in its "lowest terms."

Not only was this a finding that would endure in its viability to the students, but also its theoretical value was significant. We called it a ratio unit to signal its importance, and it became an important part of the conjecture. In the past, other researchers concentrated on students "building up" the littlest recipe to make larger batches (Streefland, 1991). This examination of ratio focuses mostly on the additive component of ratio building. What we had found was that students were willing and able to "scale down" a large recipe to its smallest, whole number equivalent. This process focuses more on the multiplicative, scaling down aspect of ratio, rather than on the more additive process of building up.

As important as the expression of these perspectives of the students was, it also was essential to consider the times when we revised and or modified our curricular interventions in

order to reflect the students' proposals during these periods of preliminary data analysis. An example from the activities used to teach decimals illustrates this point.

The first activity used in our decimal unit, the Weights Problem, was intended to be a transitional problem that reviewed ratio concepts and, at the same time, laid a foundation for decimal notation. In the first part of the Weights Problem, students were given four shapes (sticks, rectangles, circles, and blobs) made of similar materials (light wood or plastic) and were asked to ascertain the relationships in weight between the shapes. They were given the materials to construct a pan balance to uncover these relationships. By design, the weights of the shapes had a multiplicative relationship with each other (e.g., the weight of two circles equaled the weight of one stick; the weight of four sticks was equivalent to the weight of one blob; the weight of three blobs equaled the weight of four rectangles). The students were told that these shapes made up the weight system of a fictional country. When they had determined all of the relationships in this weight system, they had to create a tourist guide that explained the weight system to visitors.

Originally, we envisioned a transition to decimals by introducing a weight system that involved an increasing set of 10-to-1 relations. However, two considerations encouraged us to revise this plan. First, to understand their weight systems fully, it was clear that the students needed and desired an opportunity to use them. Second, we wanted this particular activity to lead them to reexamine the relationships among the weights in a specific system and to begin to think about how to create a notation and a set of operations in that system. We had seen previously the necessity of thinking hard about how to build a numeration system when we developed the operation of the multiplication of rational numbers. However, with the introduction of decimals, we encountered ordering, place value, ratio relations, addition, and subtraction, all combining to produce decimal notation and a new kind of number. Such a conceptualization was in marked contrast with other researchers who deal solely with issues of ordering in decimals and concern themselves only with the contrasts between decimal and whole number ordering (L. B. Resnick et al., 1989).

Our conjecture about the importance of ratio was giving us a different perspective on decimals. This led us to create the following tasks. Once the students became familiar with the weight system and made their tourist guides, they were given a bag containing different kinds of candy bars. They were asked to weigh each type of candy in the weight system and come up with a type of notation for recording the weight of each of the candies—what we called a receipt for the candy weights. Then they were asked to total up the values on the receipt and to weigh all of the candies together. This led them into a feedback system in which they expected

the two results to confirm each other. It was a powerful way to strengthen their awareness of a numeration system. Developing this task is a clear example of how analyzing preliminary data can lead to changes in both the curriculum and the conjecture. The curricular changes were substantial; on the other hand, the core of the conjecture was elaborated rather than altered.

Final Data Analysis

The second type of analysis occurs after the classroom activity has finished. The research team then plunges into the pile of data it has collected to begin the more indepth analysis of what occurred during the intervention. The methods of analysis used will depend on the methods used to collect the data, the results of the preliminary data analysis, and, of course, the conjecture itself. If more quantitative testing methods are used, some kinds of statistical analysis may be appropriate. To analyze the qualitative data, processes will be needed to organize it and then analyze its content for emergent patterns and themes. There are many ways in which to structure such a process. One could use matrix organization systems recommended by Miles and Huberman (1984) or a grounded theory approach described by A. Strauss and Corbin (1990). Patton (1990) discussed, among other analytic techniques, the process of coding and creating a category system for analyzing qualitative data.

Whatever analytic methods are used, this final stage is the slowest and most time-consuming. It is in this stage that the research team returns to the data to attempt to construct a coherent story of the development of the students' ideas and their connection to the conjecture. Once this story has been constructed, decisions about how it will be publicized and told need to be made. These decisions are discussed later.

In the splitting experiment, we used a variety of techniques to conduct the final analysis of the data. One of the unique aspects of the splitting experiment was that it was designed to accommodate multiple graduate students' theses projects. Several of the graduate students on the research team chose an aspect of the splitting experiment as the topic of their thesis. For instance, one graduate student examined the decimal unit for her master's thesis (Lachance, 1996), whereas another graduate student investigated students' use and development of tools (Preyer, 1996) throughout the last year of the intervention. In these cases, the graduate students tended to take the lead in the final analysis of the data. However, their results and tentative conclusions were always subjected to review, discussion, and refinement by the larger research group.

Each type of data collection required a different set of analytic techniques. For example, sometimes, written assessments were looked at in purely quantitative terms. How many items did the students complete correctly? How did the class fare overall? How did different groups of students (e.g., girls vs. boys) perform in comparison to each other? However, these assessments were also subject to content analysis. What types of methods did the students use to solve problems? How do we categorize those methods? What types of evidence of the students' thinking are present in these solutions?

One of the most time-consuming aspects of our final analysis was sifting through the videotapes. Generally, one or two of us viewed them and tried to code various sections. Like the experiment itself, our coding system changed as it developed. We kept notes describing the various groupings and categories of videotaped interactions. These categories grew and changed as we incorporated more data. In the end, we came up with a system of categories for describing the data. Because such research allows for these categories to emerge, they are important products of the work too.

For instance, in making sense of the decimal unit, four major categories described much of the videotaped data:

- *Precursors*: episodes in which students seemed to be developing precursors for decimal notation.
- *Concrete referents*: episodes where students used concrete referents as a means of understanding decimal notation.
- *Ratio reasoning*: episodes in which students appeared to use ratios as a means of making sense of decimals.
- *Mathematical connections*: episodes where students, attempting to make sense of a new idea, were relating various mathematical constructions to each other.

The development of these categories was also dependent on review and revision by the larger research team (see Lachance (1996) for a description of how these categories were linked to her analysis of the relation between decimals and ratio reasoning).

By the end of the study, a much more elaborated articulation of the conjecture was proposed. The elaborated conjecture can be communicated by means of examples from the research, but it should be analyzed for the robustness of its structure and connections as well. A model of the elaborated splitting conjecture is reproduced in FIG. 10.3. the reader may refer to Confrey (1996) for a fuller explanation of its contents.

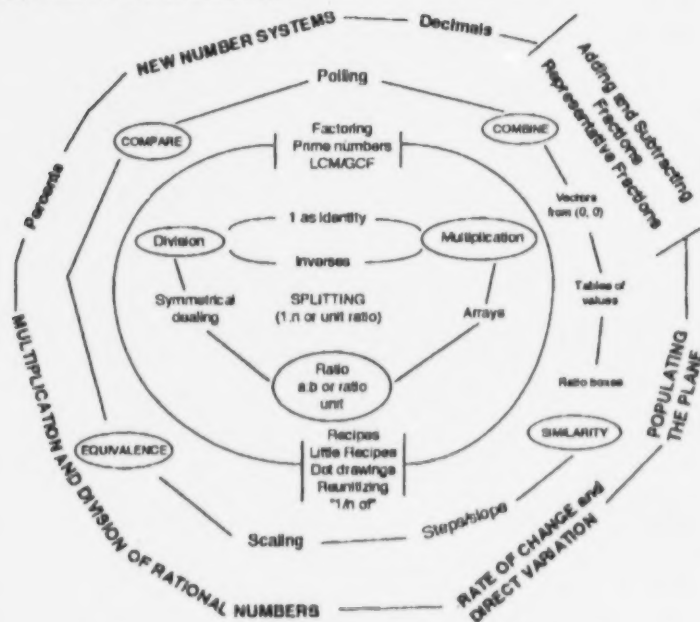


FIG. 10.3. A map of the "splitting world."

PRODUCTS OF TRANSFORMATIVE AND CONJECTURE-DRIVEN TEACHING EXPERIMENTS

An outline of the potential products of the transformative and conjecture-driven teaching experiment should be included in the research design's overall plan. Such products should be consistent with the rest of the design of the research and, thus, must take a variety of forms. Some researchers argue that the obligation of research is only to contribute to the knowledge base of the community of researchers, leaving others to interpret and translate the findings for practitioners. Consistent with our reform model, we take issue with such a narrow definition of the products of research. Producing multiple products of varied shape and structure will ensure that the conversation about the conjecture will be carried on by a diversity of audiences who will contribute to its further elaboration. It also ensures that the dialectical relation between conjecture and instruction carried out in the experiment is reproduced between the practitioner and the researcher as research results are prepared.

Potential products from this research include:

- Research papers on topics related to the conjecture such as its relationship to particular conceptual development, its forms of representations, the types of classroom interactions it promotes, and the associated outcomes of student assessments.
- Curricular materials, including software and multimedia products, that are informed by the research's findings.
- Professional development materials, such as videotapes of students' and teachers' work.
- Implementation studies looking at how the results are put into effect in various settings.
- Policy briefs for school personnel. These might be most in the school and school district where the research took place.

EVALUATING THE QUALITY OF A TRANSFORMATIVE AND CONJECTURE-DRIVEN TEACHING EXPERIMENT

This approach to research design combines multiple elements: evolving conjectures, shifts in curricula, external instructors, and new forms of assessment. Finally elements are not found in "conventional" educational studies. Therefore, a legitimate question is: Is this merely an exercise in design, notoriously serendipitous, or are there well-defined criteria for judging the adequacy and quality of such studies? Although we acknowledge that design is indeed a significant element in this work, we argue that what makes this type of work a worthwhile and significant research model is that the design element is regulated by an explicitly stated and well-developed conjecture. The burden is on the researcher to demonstrate that the conjecture guides the design of the classroom practice and the analysis of the data.

It follows that a concern with this type of guided research is *how* it is guided. Is the conjecture's development truly guided by the data or does the researcher seem to pursue an independent agenda? Because of this concern, research of this type must be subjected to a careful demonstration of its quality. However, because of the novelty and the emergent nature of this research, we cannot adopt quality standards from traditional research models and leave it at that. Such standards as internal validity, external validity, reliability, and objectivity do not seem to fit. Instead, we must identify standards that conform to the components of our model and gauge their successful application. Thus, such standards will address issues related to

ideological stance, theoretical framework, and the dialectical interaction between the conjecture and the intervention. Because the research must demonstrate its own consistency, we see one goal as ensuring the quality of the internal processes of the research. At the same time, because we acknowledge that educational research entails an intervention and aims at reform, we also must ensure quality through an assessment of the potential impacts on practice. An outline of this quality assurance plan is offered in FIG. 10.4.

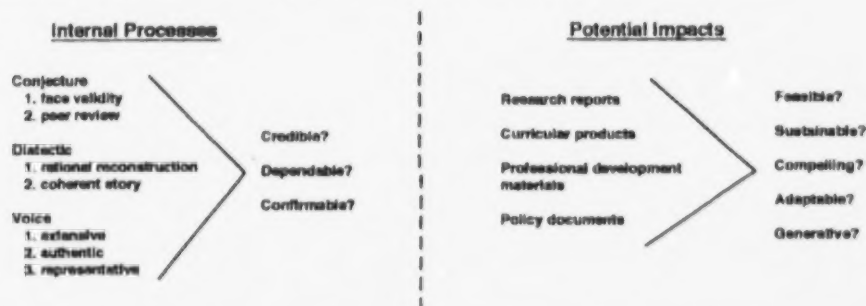


FIG. 10.4. Evaluating the transformative and conjecture-driven teaching experiment.

Ensuring the Quality of the Internal Processes

Clearly, one cannot evaluate whether all of the aspects of the research process are of high quality. However, there are some aspects that deserve special attention because of the key roles they play in both our interpretation of events and the shape and structure of the intervention itself. Perhaps the most important question about this research concerns the explanatory power of the conjecture (Robinson, 1993). Recall that one of the catalysts for creating a conjecture was a dissatisfaction with the ways in which traditional practices treat a given set of mathematical topics and the belief that such treatment prevents many students from accessing mathematical understandings. So, one of the main questions of this type of research has to be: Can the conjecture point to a better way to reconceptualize this set of topics that will allow *all students* to construct an understanding of these concepts? This question leads to the identification of three targets for evaluating the quality of the research.

First, the quality of the conjecture can be looked at as an issue of face validity in relation to peer review. At the end of the study, an audience of researchers or practitioners can evaluate the face validity of the elaborated conjecture. This will be done by an analysis of its content and its path of evolution, which will require the judgment of an expert knowledgeable in the content, and by its relationship to the literature. The researcher must provide the audiences

interested in these issues with enough background information about the conjecture, its content, pedagogy, and theoretical framework to make such an assessment in light of the research's findings.

Second, the quality of the process can be judged in relation to the coherence of the story recounting the dialectical relationship between the events in the classroom and the conjecture. How closely are the two forces in the dialectic interwoven? Does it result in a coherent story? We see this "story" as similar to Lakatos' (1976) description of his own work as a "rational reconstruction." In order to do this, the researcher will need to provide the readers with evidence of the research process, in terms of both preliminary and final data analysis.

Third, the fidelity of the study to its ideological stance can be judged. An outside audience should be able to determine whether the data allowed for the emergence of the students' voices. By this, we mean that an audience must be able to determine if the student expressions are extensive and authentic enough to convince a reader of the depth of the students' commitment to and ownership of the ideas. To do this, the researcher would need to provide ample data in the form of quotations from the students and examples of their work in discussions of the research. Along with these data, sufficient citations about the characteristics and contexts of the student-speakers, along with comments about how representative of the various student groups a given set of interactions is, must be included. Although we recognize that some students may be more articulate than others, in order to fulfill the diversity requirement, the research is obligated to demonstrate that the educational benefits have been experienced widely. In addition to the students' voices, the results of various assessment activities may support these claims. Fidelity to the ideological stance also can be gauged by whether there are concrete indicators of increased successful access to learning for all students. In terms of increased equality of outcomes, in the splitting work, we examined how the gap between better and less well prepared students changed over the course of the intervention. At the end of the 3-year splitting experiment, we found that not only did 17 of 20 students score in the 19th percentile on standardized achievement tests, but also that in contrast with comparison schools, the margin in achievement levels between better and less well prepared students had decreased significantly. In large part, this was due to the vastly improved performance of the less-well-prepared group (Scarano & Confrey, 1996).

The claims for the quality of the conjecture, of the rational reconstruction, and of the fidelity to the ideological stance are based on interpreted data. However, how does an audience know whether these data are trustworthy? Some audiences may feel that such interpreted data have been "corrupted" by the researchers' interpretations. They may ask: "How do we know

that the researcher just saw what she wanted to see or only paid attention to the data that supported her conjecture?" Such concerns speak to the issue of trustworthiness; in other words, how does an audience know that it can trust the methods and interpretation of the researcher?

To address these concerns, we turn for a moment to the field of program evaluation. Guba and Lincoln (1989) promoted a design that they called "responsive constructivist evaluation," which bears similarity to some aspects of the transformative and conjecture-driven teaching experiment. Most significantly, their evaluation model seeks to respond to the claims, concerns, and issues raised by interested parties in a given setting. As such, the design is viewed as emergent because the focus of the evaluation emerges and evolves from inputs by and negotiations among those concerned with its results. This is very similar to how we see the conjecture evolving through the interaction of students' voices and researchers' perspectives.

As a part of their proposed model for evaluation, Guba and Lincoln (1989) offered a set of criteria by which we can judge the "trustworthiness" of a given inquirer's methods and data. Because their evaluation design stresses the importance of evolving and emerging constructions and our research design has a similar focus, we have adapted several of Guba and Lincoln's notions to speak to the issue of the trustworthiness of interpreted data in conjecture-driven research. Specifically, we believe that the audiences who question the quality of interpreted data needed to be assured that these data are credible, dependable, and confirmable (Guba & Lincoln, 1989).

In terms of the credibility, a judgment of quality needs to be based on how well the constructed perspectives of the respondents (in this case, the students) matched the versions of these constructions presented by the researcher. In other words, how well did the researchers reconstruct what the students were experiencing? In several branches of social science (anthropology, sociology, etc.), specific ways of ensuring that such reconstructions are credible have been developed. Examples of them that might be used most easily in conjecture-driven research are prolonged engagement with the context of the research, persistent observation of the interactions and the setting, analysis of cases that are not in keeping with the conjecture, and checking with students to be sure that we understand what they are trying to communicate to us. By using such methods, the researcher may help to ensure that he or she is representing the constructions of the students in a credible manner.

The concept of dependability speaks to the stability of the data. For some audiences, assuring the stability of the data could be a major problem for a conjecture-driven teaching experiment where "methodological changes and shifts in constructions are expected products of an emergent design dedicated to increasingly sophisticated constructions" (Guba & Lincoln, 1989, p. 242). However, as Guba and Lincoln saw it (and we concur):

Far from being threats to dependability, such changes and shifts are hallmarks of a maturing—and successful—inquiry. But such changes and shifts need to be both tracked and traceable (publicly inspectable) so that outside reviewers . . . can explore the process, judge the decisions that were made, and understand what salient factors in the context led the [researcher] to the decisions and interpretations made" (p. 242).

Thus, to ensure dependability, researchers involved in conjecture-driven teaching experiments must keep careful track of their methodological and analytical decisions. Such evidence of the research process will allow outside parties to follow and assess the reasoning behind the methodological path taken through the data.

Finally, the construct of confirmability involves ensuring that the findings of the research are grounded in the data rather than in the whims of the research team. If the findings are confirmable, an external observer should be able to reconstruct them by way of the data. To allow this to happen, Guba and Lincoln (1989) suggested that the researcher provide an audit trail. With an audit trail, the researcher keeps evidence of the processes by which the methodological decisions were made and the data were analyzed. Then, an external auditor should be able to reconstruct the process and confirm the findings. Such an audit also can be used to ensure the dependability of the data.

Obviously, an audit is not possible in many settings because it is time consuming and requires the services of a committed individual from outside the research group. If an auditor is paid, this would entail considerable expense as well. However, in discussing research findings and presenting reports on their results, some information about what decisions were made and how and why they were made would allow audiences to have some ability to assess the dependability and confirmability of the findings.

In summary, because the quality of this research is so dependent on how well the researchers reflect upon the data and challenge themselves with respect to their evolving understandings, the question of "How do we know we can trust the researchers?" is paramount. Assuring the credibility, dependability, and confirmability of the research certainly will answer this question adequately for some audiences. However, we recognize that other audiences will

remain skeptical about the quality of this type of research. With these audiences, the researcher will have to continue to struggle with how to present compelling and convincing arguments that confirm the findings of the research.

Assessing the Potential Impact

The second part of our quality criterion is linked to our motivations for doing this type of research. One objective of this research model is to connect research and practice better so that changes in classrooms will be initiated more quickly and effectively. Thus, the second part of the quality criterion concerns how well the findings of the study are connected to achievable change. This is perhaps the most difficult criterion to meet because, in the past, bridging the gap between research results and practice has not been efficient or easy. However, because this type of research is coupled with a reform agenda, we believe that an assessment of quality must be made on its potential ability to act as a catalyst for change.

The research team needs to devise ways of making this information useful to the audiences of policymakers, teachers, parents, administrators, students, and others who initiate changes in the system. The targets for assessing the potential impacts of the research project are its products and dissemination practices discussed in the section Products of Transformative and Conjecture-Driven Teaching Experiments.

We maintain that one should expect more positive evaluations of the potential impacts on practice of a conjecture-driven teaching experiment because, unlike research designs that are isolated from classrooms, this research takes place inside them. However, such research takes place in one classroom only and, as we specified at the beginning, certain classroom constraints could be altered or relaxed. Thus, there is still a need to evaluate the potential impacts of this type of research.

Criteria for assessing these impacts include:

- *Feasibility:* Any type of research findings can be implemented if there are ample resources, both financial and human. However, schools' resources are limited, particularly at those less advantaged schools that have larger populations of at-risk students. In the quest for equity, it must be feasible for the products of research to be implemented and useful in all classrooms.
- *Sustainability:* High-quality products will have to endure and sustain their impacts over a substantial period of time. Although only time

will allow one to assess the sustainability of the impact fully, it can be explored in advance by comparing the products to similar ones.

- *Compelling Nature*: The research findings must convince practitioners of the urgency with which the proposed changes should take place. The findings and products must compel them to change. Many innovations are viewed as interesting to practitioners, but only a few create an imperative that practitioners are unlikely to ignore.
- *Adaptability*: With sensitivity to the unique and diverse contexts of schools, these products must be adaptable to a particular context. In a diverse and multicultural society with a variety of configurations of schooling, a product of research must demonstrate flexibility.
- *Generativity*: Just as it did for the research, the conjecture should provide practitioners with a powerful means of reconceptualizing a variety of classroom events, relationships, and practices. Products of research that become models for innovations will prove fruitful beyond their initial conceptualizations.

Because research has been relatively isolated from practice in the past, these criteria appear too demanding of any one team of researchers. However, as research becomes more integrated with classroom practices, we are confident that we will gain skill in assessing these criteria prior to actual implementation studies. At this point, though, implementation studies or field tests may be required in order to learn more about them.

There is no question that in our splitting work we have struggled with integrating research and practice. Finding creative ways to translate our research into practice while combating the educational system's resistance to profound change continues to be a challenge. However, we have been most successful with preparing presentations for practitioners that illustrate what students can do if given the proper activities and environment. We often bring to these sessions videotapes of students working on various projects or presenting their solutions to problems in order to demonstrate how capable and creative students are.

Although this may seem somewhat trivial compared to what we have found about the splitting conjecture, we feel it is an important effort to make. For example, whenever we talk about introducing decimals by means of the Weights Problem, we get questions that reveal the low expectations others have of students. Recall that the Weights Problem involved weights (circles, rectangles, blobs, and sticks) that have various multiplicative relationships to each other. For example, there might be a 4-to-1 relationship between the weight of circles and the

weight of blobs, and perhaps a 2-to-3 relationship between the weight of rectangles and the weight of circles. Students' work with a variety of ratios was meant to build a foundation from which they would see our decimal system as a particular set of 10-to-1 ratio relationships.

Inevitably, when we explain this problem to various groups, we are asked such questions as: "Why did you start with all those odd ratio relationships? Why didn't you make the weights all into 10-to-1 relationships? Didn't the kids get confused?" The assumption underlying these questions is that students can learn by simple straightforward methods only. Sophistication and complexity are not only unnecessary, but also may be dangerous or confusing to students, according to this line of reasoning. Having encountered these challenges frequently, we see the need to demonstrate what students are able, willing, and eager to do when given sophisticated sets of problems and how powerful their constructions are when given the chance to develop. Although they fall short of some of the changes that our findings could promote, we do believe that our presentations to teachers and peers are helping to lay the foundation for what we hope will be more transformative changes in the future.

CONCLUSION

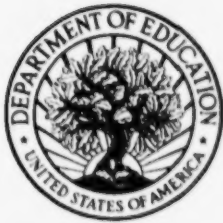
We believe that the strengths of this type of research are its unique characteristics. By listening to children and allowing theoretical understandings to evolve from what we hear, we can pave the way more effectively for increasingly equitable mathematics education practices. After all, we know that traditional models of instruction do not work for many groups of students. What better way to uncover what does work for them than to let their voices guide our emerging conjectures and theories?

The impacts that such a process can have are multiple:

- Such research is a means of articulating clearly and specifically how we want to reconceptualize mathematics curricula so that they are accessible to all students.
- By combining some of the limitations of classroom practices with the theoretical underpinnings of educational research, this model helps to build a stronger bridge between research and practice. Such a bridge can lead to a more efficient and effective reform movement.

- By making others aware of what it is possible to accomplish with children, this research model can inform decision making about what can and should be going on in mathematics education.

We believe it is vital that mathematics education research be committed to equity. By framing our research with this commitment, we are aiming for a high water mark. Intellectually interesting findings are no longer enough. Today, the worth of our work must be judged on the extent to which it improves access to learning for all of our children.



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